LQR

$K = ?$

minimizing

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt$$

① Ricatta equ

$$PA + A^T P - PBR^{-1}B^T P + Q = 0$$

② $K = R^{-1} B^T P$ $\qquad$ $u = -Kx$

Ex:

$$\dot{x} = Ax + Bu$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad R = 1 \quad A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} A & B \\ B & C \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ B & C \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} A & B \\ B & C \end{bmatrix} - \begin{bmatrix} A & B \\ B & C \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} 1 \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} A & B \\ B & C \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

note: use the solution that makes P positive definite.

Ex:

$$\dot{x} = Ax + Bu$$

$$x \in \mathbb{R},\ u \in \mathbb{R},\ b = 1$$

$$J = \int_0^\infty (q x^2 + r u^2)\, dt$$

$Q = q \in \mathbb{R} \quad r \in \mathbb{R}$

$r > 0 \quad q \geq 0$

$x^T Q x = q x^2$

① $2pa - \frac{p^2}{r} + q = 0$ } Ricatta equation

$$\boxed{\frac{p}{r} = a + \sqrt{a^2 + \frac{q}{r}}} = K$$

$$J = \int_0^\infty (x^2 q + u^2 r)\, dt$$

$$= \int_0^\infty (x^2 q + (-Kx)^2 r)\, dt$$

$$= \int_0^\infty (q + K^2 r)\, x^2\, dt$$

with state feedback, the closed loop system becomes

$$\dot{x} = -\sqrt{a^2 + \frac{a}{r}} \Rightarrow x(t) = e^{-\sqrt{a^2 + \frac{a}{r}}\, t}\, x(0)$$

note: end of first part of course.

DIGITAL SYSTEMS / DISCRETE TIME

difference equation...

$$x(kT) = f(x(k+1)T)$$

meaning: the signal depends on the last output.